NAG C Library Function Document

nag_dpotrs (f07fec)

1 Purpose

nag_dpotrs (f07fec) solves a real symmetric positive-definite system of linear equations with multiple right-hand sides, $AX = B$, where A has been factorized by nag_dpotrf (f07fdc).

2 Specification

```
void nag_dpotrs (Nag_OrderType order, Nag_UploType uplo, Integer n, Integer nrhs,
      const double a[], Integer pda, double b[], Integer pdb, NagError *fail)
```

3 Description

To solve a real symmetric positive-definite system of linear equations $AX = B$, this function must be preceded by a call to nag_dpotrf (f07fdc) which computes the Cholesky factorization of A. The solution X is computed by forward and backward substitution.

If **uplo** = **Nag_Upper**, $A = U^T U$, where U is upper triangular; the solution X is computed by solving $U^T Y = B$ and then $UX = Y$.

If **uplo** = **Nag_Lower**, $A = LL^T$, where L is lower triangular; the solution X is computed by solving $LY = B$ and then $L^T X = Y$.

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates whether A has been factorized as $U^T U$ or LL^T, as follows:

if **uplo** = **Nag_Upper**, then $A = U^T U$, where U is upper triangular;

if **uplo** = **Nag_Lower**, then $A = LL^T$, where L is lower triangular.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: **n** ≥ 0.

4: **nrhs** – Integer *Input*

On entry: r, the number of right-hand sides.

Constraint: **nrhs** ≥ 0.

5: **a**[dim] – const double *Input*

Note: the dimension, dim, of the array **a** must be at least max$(1, \textbf{pda} \times \textbf{n})$.

On entry: the Cholesky factor of A, as returned by nag_dpotrf (f07fdc).

6: **pda** – Integer *Input*

On entry: the stride separating row or column elements (depending on the value of **order**) of the matrix in the array **a**.

Constraint: $\textbf{pda} \geq \max(1, \textbf{n})$.

7: **b**[dim] – double *Input/Output*

Note: the dimension, dim, of the array **b** must be at least $\max(1, \textbf{pdb} \times \textbf{nrhs})$ when **order** = **Nag_ColMajor** and at least $\max(1, \textbf{pdb} \times \textbf{n})$ when **order** = **Nag_RowMajor**.

If **order** = **Nag_ColMajor**, the (i, j)th element of the matrix B is stored in $\mathbf{b}[(j-1) \times \textbf{pdb} + i - 1]$ and if **order** = **Nag_RowMajor**, the (i, j)th element of the matrix B is stored in $\mathbf{b}[(i-1) \times \textbf{pdb} + j - 1]$.

On entry: the n by r right-hand side matrix B.

On exit: the n by r solution matrix X.

8: **pdb** – Integer *Input*

On entry: the stride separating matrix row or column elements (depending on the value of **order**) in the array **b**.

Constraints:

if **order** = **Nag_ColMajor**, $\textbf{pdb} \geq \max(1, \textbf{n})$;
if **order** = **Nag_RowMajor**, $\textbf{pdb} \geq \max(1, \textbf{nrhs})$.

9: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: $\textbf{n} \geq 0$.

On entry, **nrhs** = ⟨*value*⟩.
Constraint: $\textbf{nrhs} \geq 0$.

On entry, **pda** = ⟨*value*⟩.
Constraint: $\textbf{pda} > 0$.

On entry, **pdb** = ⟨*value*⟩.
Constraint: $\textbf{pdb} > 0$.

NE_INT_2

On entry, **pda** = ⟨*value*⟩, **n** = ⟨*value*⟩.
Constraint: $\textbf{pda} \geq \max(1, \textbf{n})$.

On entry, **pdb** = ⟨*value*⟩, **n** = ⟨*value*⟩.
Constraint: $\textbf{pdb} \geq \max(1, \textbf{n})$.

On entry, **pdb** = ⟨*value*⟩, **nrhs** = ⟨*value*⟩.
Constraint: $\textbf{pdb} \geq \max(1, \textbf{nrhs})$.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

For each right-hand side vector b, the computed solution x is the exact solution of a perturbed system of equations $(A+E)x=b$, where

if **uplo** = **Nag_Upper**, $|E| \leq c(n)\epsilon|U^T|\,|U|$;

if **uplo** = **Nag_Lower**, $|E| \leq c(n)\epsilon|L|\,|L^T|$,

$c(n)$ is a modest linear function of n, and ϵ is the ***machine precision***.

If $\hat{x}$ is the true solution, then the computed solution x satisfies a forward error bound of the form

$$\frac{\|x-\hat{x}\|_\infty}{\|x\|_\infty} \leq c(n)\,\mathrm{cond}(A,x)\epsilon$$

where $\mathrm{cond}(A,x) = \||A^{-1}|\,|A|\,|x|\|_\infty/\|x\|_\infty \leq \mathrm{cond}(A) = \||A^{-1}|\,|A|\|_\infty \leq \kappa_\infty(A)$. Note that $\mathrm{cond}(A,x)$ can be much smaller than $\mathrm{cond}(A)$.

Forward and backward error bounds can be computed by calling nag_dporfs (f07fhc), and an estimate for $\kappa_\infty(A)$ $(=\kappa_1(A))$ can be obtained by calling nag_dpocon (f07fgc).

8 Further Comments

The total number of floating-point operations is approximately $2n^2r$.

This function may be followed by a call to nag_dporfs (f07fhc) to refine the solution and return an error estimate.

The complex analogue of this function is nag_zpotrs (f07fsc).

9 Example

To solve the system of equations $AX=B$, where

$$A = \begin{pmatrix} 4.16 & -3.12 & 0.56 & -0.10 \\ -3.12 & 5.03 & -0.83 & 1.18 \\ 0.56 & -0.83 & 0.76 & 0.34 \\ -0.10 & 1.18 & 0.34 & 1.18 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} 8.70 & 8.30 \\ -13.35 & 2.13 \\ 1.89 & 1.61 \\ -4.14 & 5.00 \end{pmatrix}.$$

Here A is symmetric positive-definite and must first be factorized by nag_dpotrf (f07fdc).

9.1 Program Text

```
/* nag_dpotrs (f07fec) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
```

```
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf07.h>
#include <nagx04.h>

int main(void)
{
  /* Scalars */
  Integer  i, j, n, nrhs, pda, pdb;
  Integer  exit_status=0;
  NagError fail;
  Nag_OrderType order;
  /* Arrays */
  char   uplo[2];
  double *a=0, *b=0;
  Nag_UploType uplo_enum;

#ifdef NAG_COLUMN_MAJOR
#define A(I,J) a[(J-1)*pda + I - 1]
#define B(I,J) b[(J-1)*pdb + I - 1]
  order = Nag_ColMajor;
#else
#define A(I,J) a[(I-1)*pda + J - 1]
#define B(I,J) b[(I-1)*pdb + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f07fec Example Program Results\n\n");
  /* Skip heading in data file */
  Vscanf("%*[^\n] ");
  Vscanf("%ld%ld%*[^\n] ", &n, &nrhs);
#ifdef NAG_COLUMN_MAJOR
  pda = n;
  pdb = n;
#else
  pda = n;
  pdb = nrhs;
#endif
  /* Allocate memory */
  if ( !(a = NAG_ALLOC(n * n, double)) ||
       !(b = NAG_ALLOC(n * nrhs, double)) )
    {
      Vprintf("Allocation failure\n");
      exit_status = -1;
      goto END;
    }

  /* Read A and B from data file */
  Vscanf(" ' %1s '%*[^\n] ", uplo);
  if (*(unsigned char *)uplo == 'L')
    uplo_enum = Nag_Lower;
  else if (*(unsigned char *)uplo == 'U')
    uplo_enum = Nag_Upper;
  else
   {
     Vprintf("Unrecognised character for Nag_UploType type\n");
     exit_status = -1;
     goto END;
   }

  if (uplo_enum == Nag_Upper)
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = i; j <= n; ++j)
            Vscanf("%lf", &A(i,j));
        }
      Vscanf("%*[^\n] ");
    }
  else
```

```
    {
      for (i = 1; i <= n; ++i)
        {
          for (j = 1; j <= i; ++j)
            Vscanf("%lf", &A(i,j));
        }
      Vscanf("%*[^\n] ");
    }

  for (i = 1; i <= n; ++i)
    {

      for (j = 1; j <= nrhs; ++j)
        Vscanf("%lf", &B(i,j));
    }
  Vscanf("%*[^\n] ");

  /* Factorize A */
  f07fdc(order, uplo_enum, n, a, pda, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07fdc.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Compute solution */
  f07fec(order, uplo_enum, n, nrhs, a, pda, b, pdb, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from f07fec.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
  /* Print solution */
  x04cac(order, Nag_GeneralMatrix, Nag_NonUnitDiag, n, nrhs, b, pdb,
         "Solution(s)", 0, &fail);
  if (fail.code != NE_NOERROR)
    {
      Vprintf("Error from x04cac.\n%s\n", fail.message);
      exit_status = 1;
      goto END;
    }
 END:
  if (a) NAG_FREE(a);
  if (b) NAG_FREE(b);
  return exit_status;
}
```

9.2 Program Data

```
f07fec Example Program Data
  4  2                        :Values of N and NRHS
  'L'                         :Value of UPLO
  4.16
 -3.12   5.03
  0.56  -0.83   0.76
 -0.10   1.18   0.34   1.18   :End of matrix A
  8.70   8.30
-13.35   2.13
  1.89   1.61
 -4.14   5.00                 :End of matrix B
```

9.3 Program Results

```
f07fec Example Program Results

 Solution(s)
             1           2
 1      1.0000      4.0000
 2     -1.0000      3.0000
 3      2.0000      2.0000
 4     -3.0000      1.0000
```